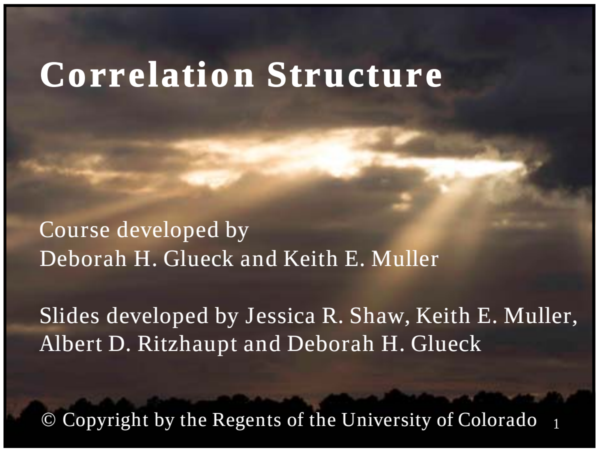


Learning Objectives

Review correlation concepts.

Define standard deviation and variance.

Define correlation structure.

Describe how correlation structure influences power and data analysis.

2

Learning Objectives

Define compound symmetry.

Describe how clusters of observations induce compound symmetric correlation structures.

3

Learning Objectives

Describe that multivariate study designs can induce a variety of different correlation structures.

Review how multivariate designs influence correlation structure.

4

For accurate power analysis, the expected correlation structure of observations must be summarized

Correlation structure summarizes the correlation between pairs of observations.

5

We will discuss correlation structures induced by multilevel and longitudinal studies

Clustering and repeated measures both influence correlation structure.

6

Specification of the correlation structure is critical to achieve correct power and sample size calculation

Correct correlation structure
⇩
Accurate sample size

Incorrect correlation structure
⇩
Inaccurate sample size
⇩
Ethical dilemmas

Glueck, D.H.

7

REVIEW: CORRELATION CONCEPTS

8

Recall, correlation is a measure of the relationship between two variables

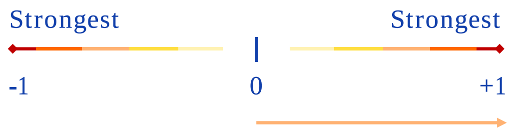
Correlation "indicates the strength and direction of the relationship between two random variables."

Rosner, 2010, p. 127

9

Numbers further from zero represent stronger correlation

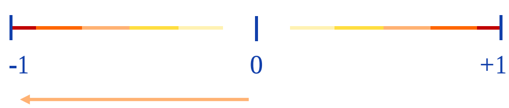
Positive correlation (between 0 and 1) indicates that variables change in the same direction.



10

Correlation (r) ranges from -1 to 1

Negative correlation (between -1 and 0) indicates that two variables change in opposite directions.



11

A correlation of zero indicates that two variables are unrelated

Variables with correlation equal to zero have rates and directions of change that are uncorrelated, or unrelated.

12

There is a relationship between correlation and r^2

A correlation of 1 or -1 corresponds to an r^2 of 1, which means there is perfect prediction.

A correlation of zero corresponds to an r^2 of zero, which means that the model explains none of the variability.

13

Variables that are independent always have correlation equal to zero

Mathematically speaking, variables with correlation equal to zero are not necessarily independent.

For the purposes of this class, we will focus on cases where correlation of zero indicates that two variables are independent.

14

A correlation matrix is a concise way of summarizing patterns of correlation

Correlation

Variable	1	2	3	4
1	1.0	0.8	0.5	0.1
2	0.8	1.0	0.1	0.5
3	0.5	0.1	1.0	0.8
4	0.1	0.5	0.8	1.0

15

The correlation between a variable and itself is always +1.0

Correlation

Variable	1	2	3	4
1	1.0	0.8	0.5	0.1
2	0.8	1.0	0.1	0.5
3	0.5	0.1	1.0	0.8
4	0.1	0.5	0.8	1.0

Glueck, D.H.

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WORKING WITH COVARIANCE

17

Covariance and correlation are related to each other by a simple equation

The covariance of variables X and Y is

$$c_{xy} = r_{xy} s_x s_y$$

where r_{xy} is the correlation coefficient between x and y, s_x is the standard deviation of x, and s_y is the standard deviation of y.

18

One can convert from correlation and standard deviations to covariance and variances

Knowing the information $\{r_{xy}, s_x, s_y\}$ allows computing the covariance matrix.

The covariance matrix contains individual variances on the diagonal.

Therefore, knowing a covariance matrix allows computing $\{r_{xy}, s_x, s_y\}$.

19

CORRELATION STRUCTURE OF CLUSTERS

20

Clustering within a level in a study induces a correlation structure called compound symmetry

Recall that levels, sometimes referred to as groups or clusters, share similarities which induce correlation.

21

Compound symmetric correlation structures exhibit two notable features

1. All independent sampling units have the same standard deviation.
2. The correlation between any two independent sampling units is the same, no matter which two are chosen.

22

Compound symmetric correlation structures exhibit two symmetries

Correlation

ISU	1	2	3	4
1	1.0	0.5	0.5	0.5
2	0.5	1.0	0.5	0.5
3	0.5	0.5	1.0	0.5
4	0.5	0.5	0.5	1.0

Variance

ISU	σ^2
1	0.3
2	0.3
3	0.3
4	0.3

23

CORRELATION STRUCTURE OF REPEATED MEASURES

24

The pattern of correlation between repeated measures is usually different than the pattern within clusters

Observations in longitudinal studies typically do not follow a compound symmetric pattern of correlation.

25

Longitudinal studies induce correlation between measurements

Measurements from the same person taken at two or more times will be correlated.

Longitudinal measures are a special case of repeated measures.

Rosner, 2010

26

The variability of observations may change over the course of repeated measures designs

Example:

The standard deviation in the outcome observed at measurement one may be different from the standard deviation observed at measurement two.

27

Similarly, correlation may change over the course of repeated measures designs

Example:

The correlation between measurements one and two typically will be higher than the correlation between measurements one and four.

28

The LEAR correlation model is useful for many repeated measures designs

LEAR stands for Linear Exponential Autoregressive.

The LEAR model has a base correlation and a decay parameter that controls how fast correlation diminishes over time.

29

Here is a LEAR Example with base correlation=0.7 and decay parameter=0.4

Correlation

Time	1	2	3	4
1	1.	0.70	0.65	0.61
2	0.70	1.0	0.70	0.65
3	0.65	0.70	1.0	0.70
4	0.61	0.65	0.70	1.0

30

In studies with spatial repeated measures, a variable is measured repeatedly over space

Spatial repeated measures designs have complex correlation structures.

31

Previous data should be used to model the correlation in spatial repeated measures designs

Correlation may decrease as distance increases.

32

Multivariate repeated measures studies require similarly complex correlation structures

In multivariate designs, correlation is the result of multiple response variables being measured for each independent sampling unit.

33

Correlation structures for multivariate repeated measures studies should also be based on previous data

The inter-relationship of multiple response variables may be complex.

34

Modeling correlation structures is made easier by existing technology

Software simplifies the implementation of complex models.

A user specifies a model for each level and then the software combines levels to generate complete model.

35

REVIEW OF LEARNING OBJECTIVES

36

True or false? Nature can determine covariance structures

TRUE

FALSE

37

True or false? It is sometimes okay to ignore correlation within clusters

TRUE

FALSE

38

True or false? Organizational structures can affect correlation

TRUE

FALSE

39


Questions?
40